Deep Functional Dictionaries: Learning Consistent Semantic Structures on 3D Models from Functions

Introduction

Various 3D semantic attributes can be encoded as **per-point probe functions** on 3D geometries.









Keypoints

Given a collection of related 3D shapes $\{X_i\}$, we consider how to jointly analyze such probe functions over different shapes $\{f_i\}$, and how to discover common latent structures using a neural network - even in the absence of any correspondence information. Our network produces a small **dictionary** of basis functions for each shape $A(\mathcal{X}_i; \Theta)$, a dictionary whose linear span includes the semantic functions provided for that shape. Even though our shapes have independent discretizations and no functional correspondences are provided, the network is able to generate latent bases, in a consistent order, that reflect the shared semantic structure among the shapes.

- Each shape \mathcal{X} is given as n **points** sampled on its surface.
- A function f is represented with a vector in \mathbb{R}^n (a scalar) per point).
- The atoms of the dictionary $A(\mathcal{X}; \Theta) \in \mathbb{R}^{n \times k}$ are represented as **columns** of a matrix. k is a large number for the maximum size of the dictionary.



• The columns of $A(\mathcal{X}; \Theta)$ are enforced to encode **atomic** semantics in applications (e.g. atomic instances in segmentation) by adding appropriate constraints.

Deep Functional Dictionary Learning Framework

Our neural network takes shape-function pairs (\mathcal{X}, f) as inputs in training, and outputs a functional dictionary $A(\mathcal{X}; \Theta)$ for each shape. Loss function $L(A(\mathcal{X}; \Theta); f)$ is designed to minimize 1) the **projec**tion error from f to the vector space $A(\mathcal{X}; \Theta)$, and 2) the number of atoms in the dictionary matrix:

> $L(A(\mathcal{X};\Theta);f) = \min_{\boldsymbol{x}} F(A(\mathcal{X};\Theta),\boldsymbol{x};f) + \gamma \|A(\mathcal{X};\Theta)\|_{2,1}$ $F(A(\mathcal{X};\Theta),\boldsymbol{x};f) = \|A(\mathcal{X};\Theta)\boldsymbol{x} - f\|_2^2$ s.t. $C(A(\mathcal{X};\Theta), \boldsymbol{x})$

 $x \in \mathbb{R}^k$ is a linear combination weights, and $C(A(\mathcal{X}; \Theta), x)$ is a set of **constraints** on both $A(\mathcal{X}; \Theta)$ and x determined in each application.

Since the nested minimization $F(A(\mathcal{X};\Theta), x; f)$ is generally not solved analytically due to $C(A(\mathcal{X}; \Theta), \mathbf{x})$, we use an **alternating minimization** scheme:

function Single Step Gradient Iteration($\mathcal{X}, f, \Theta^t, \eta$) Compute: $A^t = A(\mathcal{X}; \Theta^t)$. Solve: $\boldsymbol{x}^t = \arg\min_{\boldsymbol{x}} \|A^t \boldsymbol{x} - f\|_2^2$ s.t. $C(\boldsymbol{x})$. Update: $\Theta^{t+1} = \Theta^t - \eta \nabla L(A(\mathcal{X}; \Theta^t); f, \mathbf{x}^t).$ end function

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Reflectance

Weakly-supervised Co-segmentation



[0, 1] range and **single point** constraints:



 $C_{\mathsf{key}}(A(.$

Smooth Function Apprixmation and Mapping



Unit vector constraint:

 $C_{map}(A$

Neural Network Architecture

In all experiments, we used PointNet [1] segmentation architecture without any modification, but any other architecture processing 3D geometry can be employed.

ShapeNet Semantic Part Segmentation

Ours outperforms vanilla PointNet [1] on average mean IoU metric when finding correspondences between ground truth and predicted part segments in each shape (left). Note that PointNet uses additional part label supervision. The average Mean IoU is still comparable when finding correspondences between part labels and atom indices in each category (right).





Figure 1: Colors of part segments indicate indices of atoms, which are consistent unless part geometries are not distinguishable: e.g. shade of ceiling lamp and base of standing lamp (in circles).

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Adaptation in Applications

[0, 1] range and **partitioning** constraints:

 $|\mathbb{0} \leq x \leq \mathbb{1}|$ $C_{\text{seg}}(A(\mathcal{X};\Theta), \mathbf{x}) = \{ \mathbf{0} \le A(\mathcal{X};\Theta) \le \mathbf{1} \}$ $\Sigma_j A(\mathcal{X}; \Theta)_{i,j} = 1$ for all i

Weakly-supervised Keypoint Correspondence Estimation

$$(\mathcal{X}; \Theta), \boldsymbol{x}) = \begin{cases} \mathbb{O} \leq \boldsymbol{x} \leq \mathbb{1} \\ \mathbb{O} \leq A(\mathcal{X}; \Theta) \leq \mathbb{1} \\ \Sigma_i A(\mathcal{X}; \Theta)_{i,j} = 1 \text{ for all } j \end{cases}$$

$$A(\mathcal{X};\Theta), \boldsymbol{x}) = \left\{ \sum_{i} A(\mathcal{X};\Theta)_{i,j}^{2} = 1 \text{ for all } j \right\}$$

	Ours (per shape)	Ours (per cat.)
avg. mloU	84.6	77.3



The performance of instance proposal prediction in 3D scenes is evaluated with proposal recall metric. The class confusion (middle) is calculated as frequencies being included in the same atoms.



MPI-FAUST Human Shape Bases Synchronization

Our framework can find synchronized atomic functions for which linear combination can approximate any random continuous function (left). Any information in one shape can be transferred to the other shapes through the synchronized bases: e.g. parts (right).



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References

- CVPR, 2018.





ShapeNet Keypoint Correspondences

The percentage of correct keypoints (PCK) is measured both when finding correspondences between ground truth and predicted keypoints in each shape (red line) and finding between ground truth labels and atom indices for all shapes (green line). The PCK curves are **identical**, meaning that the order of predicted keypoints is consistent across

S3DIS Instance Segmentation

shapes

[Code] https://github.com/mhsung/deep-functional-dictionaries

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[1] C. R. Qi, H. Su, K. Mo, and L. J. Guibas. Pointnet: Deep learning on point sets for 3d classification and segmentation. In CVPR, 2017. [2] W. Wang, R. Yu, Q. Huang, and U. Neumann. Sgpn: Similarity group proposal network for 3d point cloud instance segmentation. In